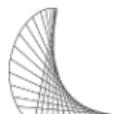


Mathematical Writing

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Introduction

Acknowledgments and disclaimer

Few (if any) of the guidelines and observations below are original. At the end of the file, there are several slides with references.

Stating rules for good mathematical writing is easier than following them, and I do not claim that the texts authored by myself should be seen as a model of particularly good mathematical writing.

Aspects of good mathematical writing

- Goal: Document a mathematical result/theory, present to others.
- Different types of mathematical texts: Thesis, Research paper, research monograph, text book, survey paper, ...
- Global vs. local structure
- How to do it: The writing process

What is in these slides

- Introduction
- The writing process
- Global structure
- Local structure
- Typography, LaTeX
- References

Fundamental principles

The task of the author is to make understanding the text as easy as possible for the reader.

- It is usually worth the effort, if the text can be improved to the intended readers.
- What is the intended/expected group of readers?
- What is the objective you want to achieve with the text?

NB: It is very likely that some time after writing your text, you will be one of your readers yourself.

Fundamental principles

Strive for

- Correctness,
- clarity,
- appropriate level of detail,
- consistency,
- appropriate level of redundancy.

Fundamental principles

Make conscious decisions.

Writing well requires a serious effort.

Honesty is the best policy (Halmos)

Note however that Proposition 3.5 does not imply that (some statement) holds.

- What does “does not imply” mean (formal-logical sense, or “does not imply easily”?)
- Why not? (Is it obvious, or does it require an argument?)
- Or does it mean that the author tried to but did not succeed in proving (some statement) using Proposition 3.5?

General criteria

How to compare different ways to write a math text?

- Shorter is better than longer,
- reaching the main result(s) faster is better,
- aesthetic satisfaction (how easy is it to give good motivation?),
- appropriate level of generality, “explanation of underlying reasons”

The writing process

Strategies for getting started ...

- top-down: lay out the rough structure of the manuscript (sections of the text; part of the introduction, stating goals and giving context, statements of main theorem(s)), or
- bottom-up: start by typing the proof of a “key lemma”, from there work your way up, adding further results etc.

Start by what you are eager to do. (Schiffer: “The best sections of a book are always those which are written enthusiastically in one piece.”)

...and for getting done

- focus on small steps,
- make a plan, set milestones,
- make writing a routine; work on your manuscript every work day.

Write in spirals (Halmos)

When to stop ...

...and call it a paper.

- There needs to be some “substance”,
- shorter papers are (all in all) easier to publish (and easier to handle for the author),
- and make for a longer list of publications,
- when a paper is published (as a preprint), others can use and improve on its results – this can have advantages and drawbacks

Proofreading

- ...usually has to be done many times.
- Leave a bit of time between rounds of proof-reading.
- Get help from other people.

How to get good at mathematical writing

- Read books, papers written by others,
- think about whether you like or dislike the way of writing, and why,
- (maybe) look at texts about good writing,
- mainly: practice writing yourself, and get feedback from others.

Types of writers

(see Mannheim workshop)

Against procrastination

- Use “quality time” for writing, put it in your daily schedule.
- Be realistic with what you will be able to do, leave time for leisure activities and rest.
- Avoid distractions (social media, mobile phone, ...),
- arrange your work place accordingly.

Writing an introduction

Purpose of the introduction?

What makes for a good introduction?

- Sell your results, your paper to people: get people interested, make them aware of your results, make them read your paper,
- Summary of results: Help people get an idea of what is in the paper, help people to actually read the paper, help people who have read the paper and are coming back to it,

Typical content of introduction

- motivation, context,
- key results (the introduction should contain at least one theorem),
- key methods used in the proofs,
- related work (by yourself and/or others),
- content of individual sections of the paper,
- acknowledgments (could also be put elsewhere).

Writing an abstract

Purpose of the abstract?

What makes for a good abstract?

Priority: short summary rather than advertisement.

Global structure

Formal vs. informal parts of the text

Clearly distinguish between them, and separate them.

Importance of informal parts

- Motivation, advertising,
- facilitate understanding of proofs,
- facilitate appreciation of results, difficulties

Linearity

The structure of a (mathematical) text is linear.

When she reaches the end of a sentence, the reader may expect that the text until this point is self-contained.

If this is not the case, it has to be communicated to the reader:

Typical examples: Theorem/Proof, "..., where X is Y .", the introduction, sometimes forward references are useful.

Locality

Allow for grasshopper readers (as much as possible).

put things where they are needed (be nice on the reader's memory capacity)

Amount of detail

Don't underexplain, don't overexplain.

Repetition – good and bad

Different styles of writing

Depending on type of text (e.g., thesis vs. journal publication)

How would you characterize the writing styles of

- Vakil, The rising sea (Foundations of Algebraic Geometry)
- Hartshorne, Algebraic Geometry,
- Grothendieck, Fondements de Géométrie Algébrique,
- Dieudonné, Grothendieck, Éléments de Géométrie Algébrique,
- De Jong et al., Stacks Project.

How to help the reader

- add motivation, explain your ideas; explain what you will be doing, what is going on (make it easy for the reader to answer the question “Why should I read on?”)
- divide text into sections / smaller segments (but not too small),
- choose descriptive (section) titles,
- use Lemma, Proposition, Theorem, Corollary consciously,
- add illustrative (counter-)examples,
- in books or other long texts: provide an index, index of notation.

Local structure

Make your text readable

It should be possible to read the text (as in reading it aloud, as grammatical sentences).

A mathematical text is a text. (But not a novel: Write clearly, relatively simply, using short sentences — your readers might not be native speakers.)

Make your text readable, continued

Do not use symbols (such as \Rightarrow , \forall , \exists) in the text, write out (most) abbreviations (iff, s. t., WLOG), write out verb forms (it is, not it's).

Not: In [1] it is proved, better: In Grothendieck's paper [1] it is proved ... (this is also nice towards the author of the cited publication, at least when the citation appears for the first time, and often saves the reader the time to look at the bibliography).

Do no overuse parentheses as part of the text.

Rarely use footnotes.

Notation

- Define all notation (at least in passing). (Exceptions?)
- Choose clear notation.
- Prefer standard notation. Let π be a real number.
- Choose meaningful/self-explanatory notation.
- Choose consistent notation.
- Choose lean notation.

When citing a paper, alert the reader of differences in the use of notation.

Often seen conflicts of notation:

- k (natural number vs. a field)
- i (natural number, closed immersion, $i \in \mathbb{C}$ imaginary unit)
- S_n
- σ
- F (Frobenius morphism vs. a field)

How to write chains of equalities

Definitions

A curve C with n distinct points p_1, \dots, p_n on it is an n -pointed curve.

‘Condition’ or ‘part of the data’?

Empty statements

For every vector space V there exists a homomorphism $V \rightarrow (V^*)^*$.

Quantifiers

Make quantifiers unambiguous.

$$\exists \delta > 0 : |x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon, \quad \forall \varepsilon > 0$$

Be careful when using “any”.

We call $p \in \mathbb{N}_{>1}$ a prime number if $a \nmid p$ for any $a \in \{2, \dots, p-1\}$.

How to help the reader

- Add some redundancy (the fundamental group $\pi_1(X)$ vanishes)

English language

Write correct English.

British vs. American English.

Write (relatively) simple English.

Omit superfluous words, filler words (this, as many of the other points, applies to other languages, too!)

Be careful with “obvious”, “trivial”, “easy to check”, “well-known”.

Be careful with judgmental language.

Mathematical meaning of “common” English words

the

simple

identify A with B

such that, so that

Relative clauses

Let G be a group of order 60, which is solvable.

Defining (or restrictive) relative clause:

Essential to determine the meaning of what the relative clause refers to. Use which or that without comma.

Non-defining (or non-restrictive) relative clause:

Additional information about the object of the relative clause. Use which or that with comma.

e.g. \neq i.e. \neq in particular

I vs. we vs. one

- Avoid the use of “I” (except possibly for personal messages),
- “we” = the author and the reader,
- “one” is possible, but rephrasing is often better: “Therefore X is quasi-compact.” instead of “Therefore one sees that X is quasi-compact.”

Typography, LaTeX

Basic typography rules

- Do not start a sentence with a mathematical symbol.
- Mathematical symbols that do not form a unit must be separated by at least one word.
- Avoid very similar looking symbols (this can depend on the font(!)).

LaTeX

Use LaTeX,

as a default, trust LaTeX in questions of layout and typography,

use `\cite` (put in the references from the beginning!),

use `\ref`, `\label`,

take LaTeX warnings into account,

number theorems, lemmas, definitions, etc. with a single counter.

use (self-defined) macros (then it is a lot easier to change notation later)

Use LaTeX properly

single letter identifiers in italics

Let n be an integer.

math operators in upright font (`\DeclareMathOperator`),

proper spacing, choice of symbols: $\langle v, w \rangle = 0$ vs. $\langle v, w \rangle = 0$,

References: The TeX book, ...

Other tools

- backups,
- git (or other versioning system), gitlab/github,
- latexdiff,
- powerful text editor,
- Overleaf, <https://www.overleaf.com>,
- Zotero, Obsidian, ...

References

Writing well in general

- W. Zinsser, *On Writing Well*
- W. Strunk, E. B. White, *The Elements of Style*
- R. Flesch, A. H. Lass, *The Classic Guide to Better Writing*
- *The Chicago Manual of Style*
- *Oxford Style, New Hart's Rules*

Math-specific books

- N. Steenrod, P. Halmos, M. Schiffer, J. Dieudonné, *How to write mathematics*, Amer. Math. Soc. (1973).
- D. Knuth, T. Larrabee, P. Roberts, *Mathematical writing*, https://jmlr.csail.mit.edu/reviewing-papers/knuth_mathematical_writing.pdf
- S. Krantz, *A primer of mathematical writing*, <https://arxiv.org/pdf/1612.04888.pdf>

...and other math-specific texts

- Clara Löh, <https://loeh.app.uni-regensburg.de/seminars/eaw.pdf>,
- F. E. Su, *Guidelines for good mathematical writing*,
<https://faculty.math.illinois.edu/~kkirkpat/good-math-writing.pdf>,
- Tom Leinster, <https://www.maths.ed.ac.uk/~tl/tips.pdf>,
- Martin Lübbecke,
<https://mluebbecke.wordpress.com/2014/11/21/how-to-write-a-paper/>,
- Mirjam Dür, <https://www.math.uni-trier.de/~duer/MathSchreiben.pdf>,
- Bjorn Poonen, <https://math.mit.edu/~poonen/papers/writing.pdf>.

Talks/slides on mathematical writing

- Dimitri Bertsekas, <https://newslab.ece.ohio-state.edu/for%20students/resources/tenrules.pdf>
- Slides for a workshop at Mannheim University Library, <https://www.bib.uni-mannheim.de/media/Einrichtungen/Universitaetsbibliothek/Dokumente/Recherche/2017-06-29-How-to-write-good-mathematical-articles.pdf>
- Jean-Pierre Serre, *How to write mathematics badly*, <https://www.youtube.com/watch?v=ECQyFzzBH1o>

Acknowledgments

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